



CNN : Cool Neural Networks

« La découverte du feu pendant un hiver froid »

Computer Vision



➤ Classification des images:



64x64

→ Cat? (0/1)

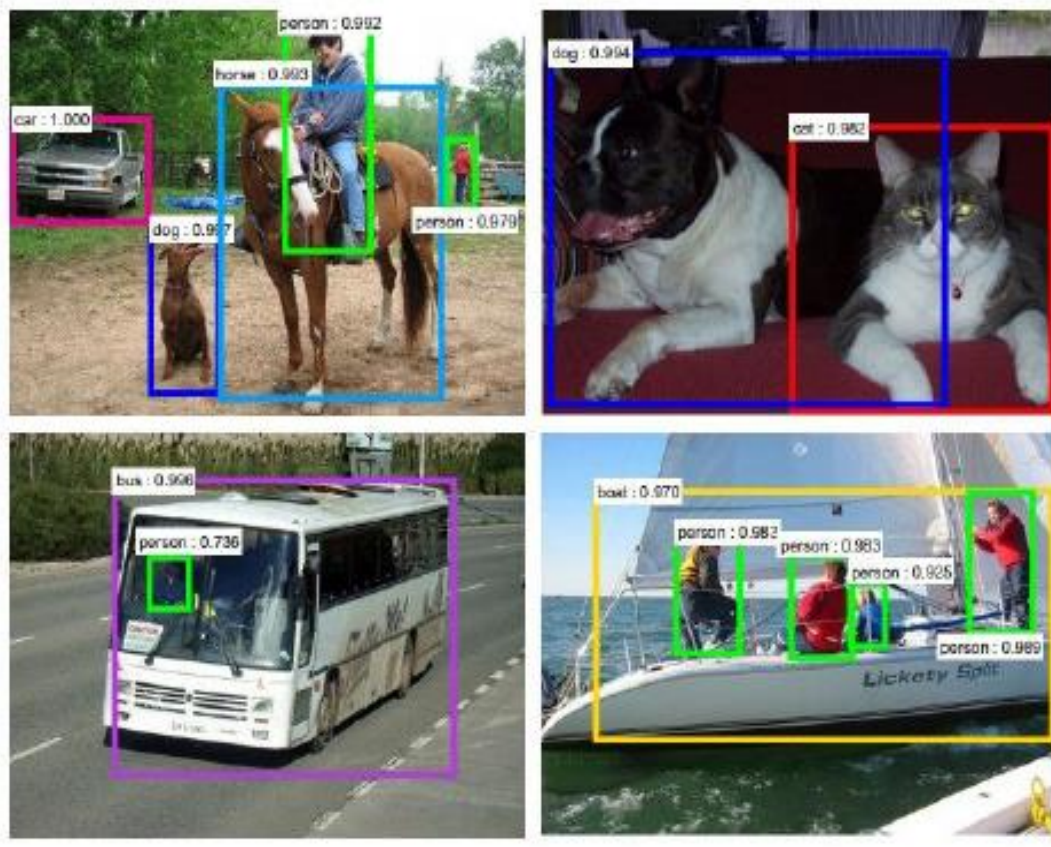
➤ Transfert du style:



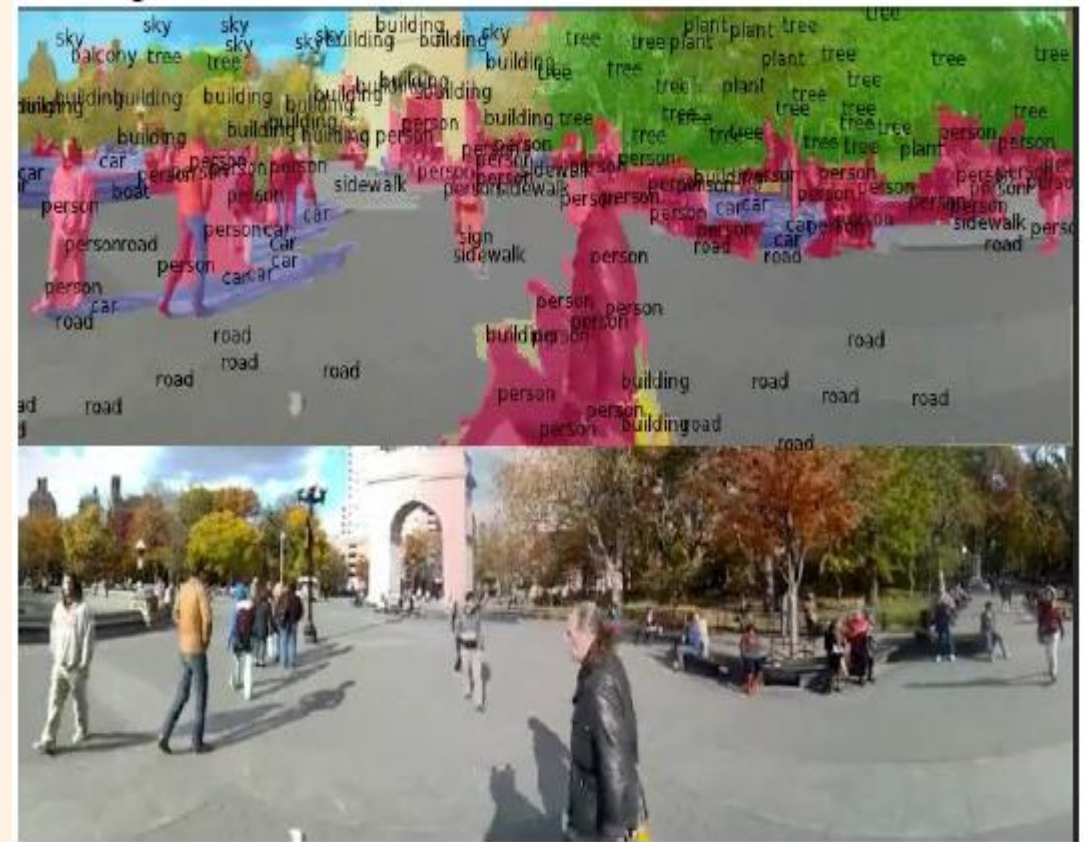
Encore plus de Computer Vision



➤ Détection d'objets:



➤ Segmentation sémantique:



Encore plus...



➤ Estimation de posture



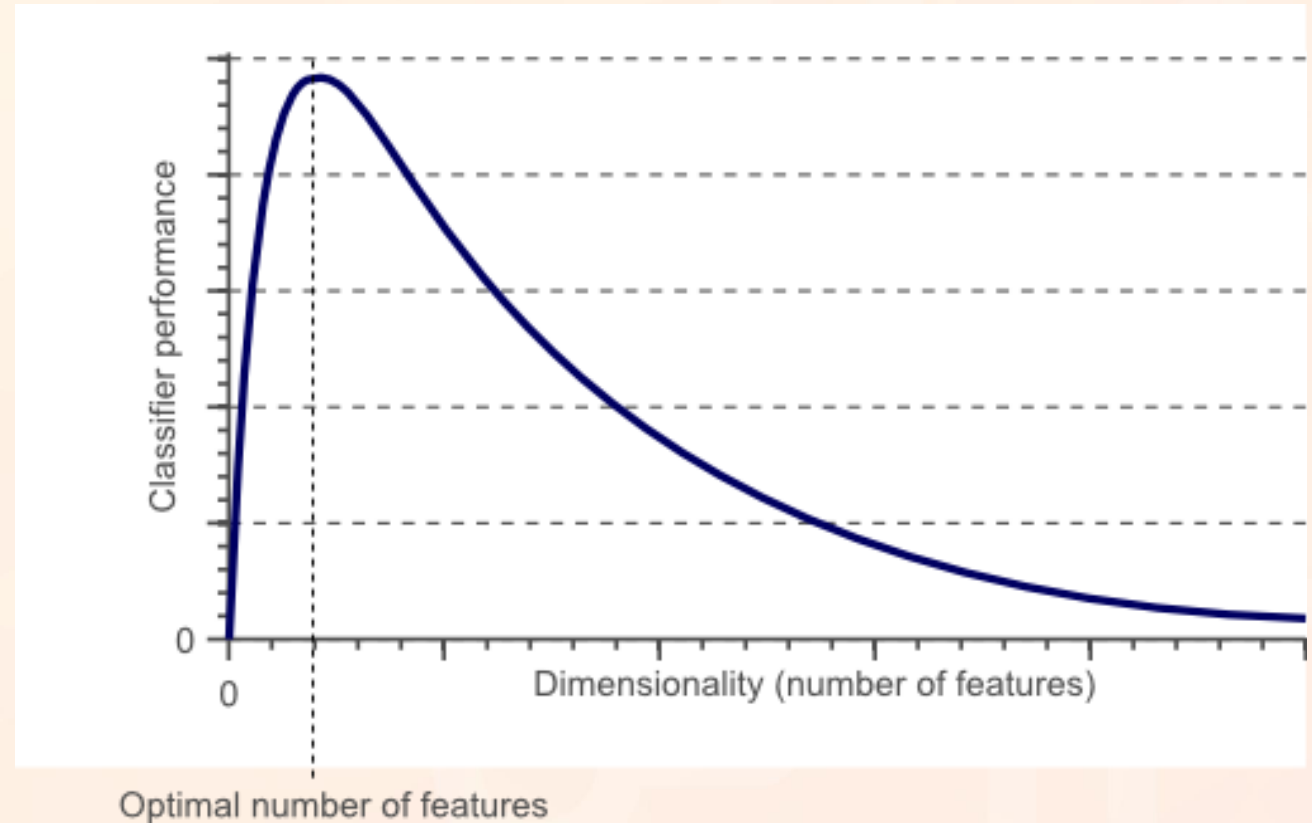
➤ Apprentissage par renforcement



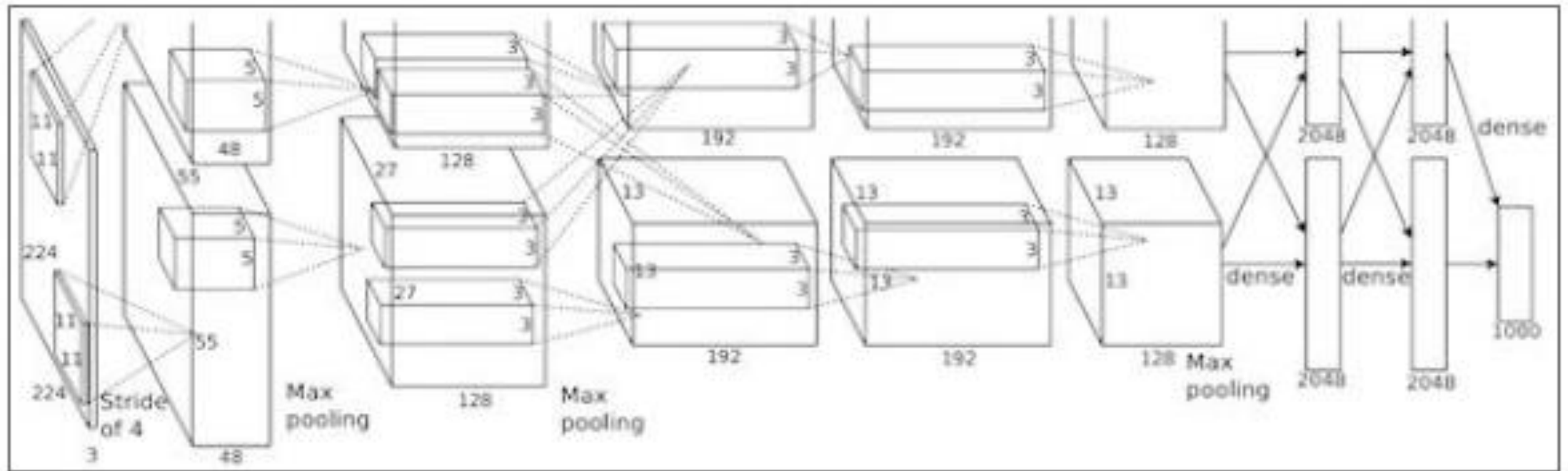
Le problème avec les MLP



1000 x 1000 x 3

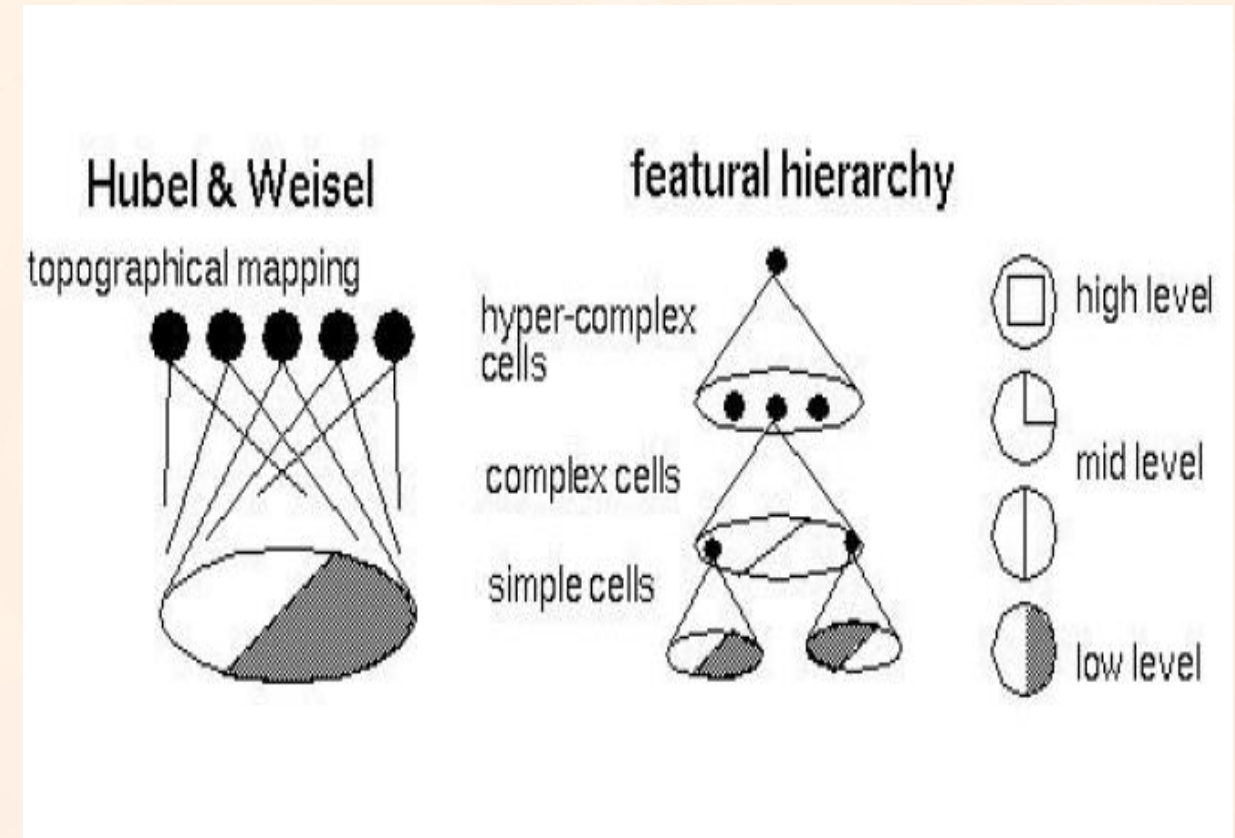
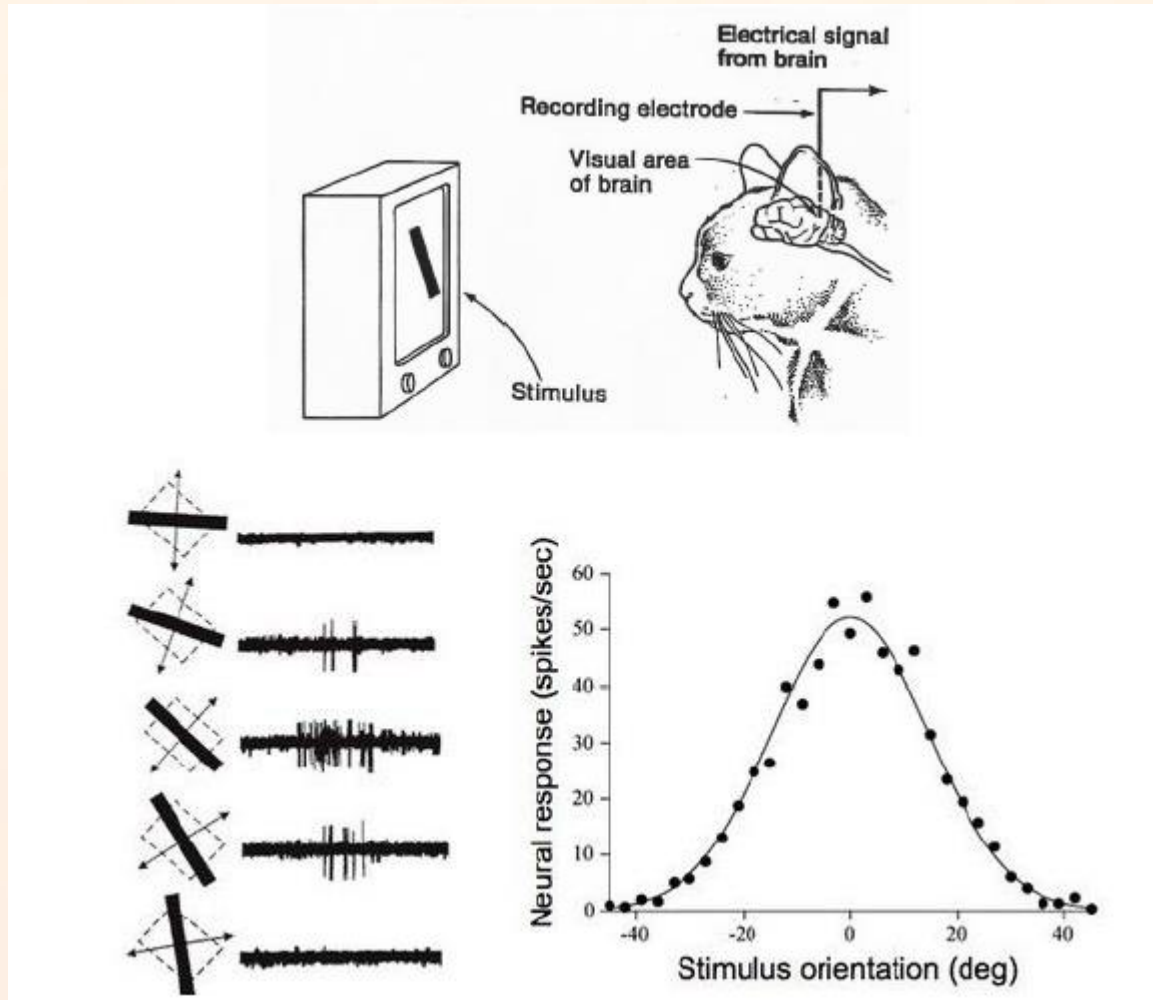


Le premier CNN: LeNet



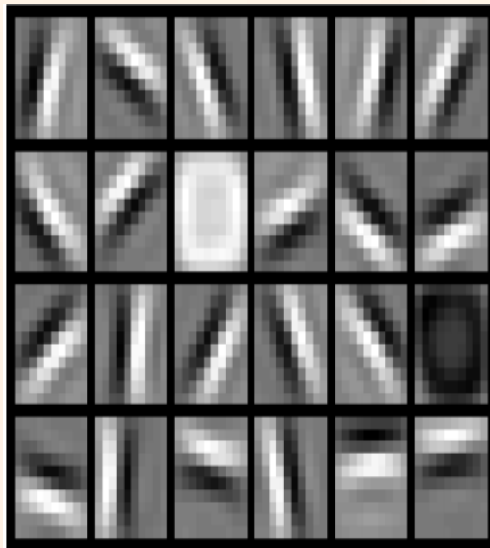
“AlexNet”

Tout a commencé avec l'histoire d'un chat



https://www.youtube.com/watch?v=8VdFf3egwfg&feature=youtu.be&t=1m10s&ab_channel=AliMoeeny

Hypothèse d'Hiérarchie des caractéristiques



Convolution



1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

5 x 5 – Image Matrix



1	0	1
0	1	0
1	0	1

3 x 3 – Filter Matrix

1 _{x1}	1 _{x0}	1 _{x1}	0	0
0 _{x0}	1 _{x1}	1 _{x0}	1	0
0 _{x1}	0 _{x0}	1 _{x1}	1	1
0	0	1	1	0
0	1	1	0	0

Image

4		

Convolved
Feature

Filtres



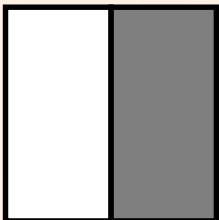
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

*

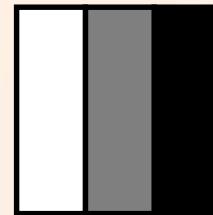
w11	w12	w13
w21	w22	w23
w31	w32	w33

=

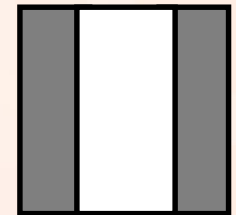
0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0



*



=

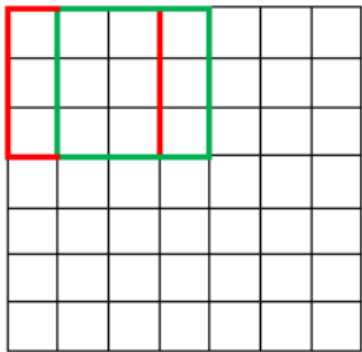


Taille de noyau, Stride et Padding

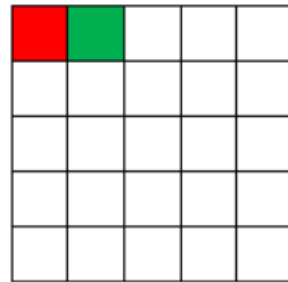


Stride de (1, 1) = 1

7 x 7 Input Volume

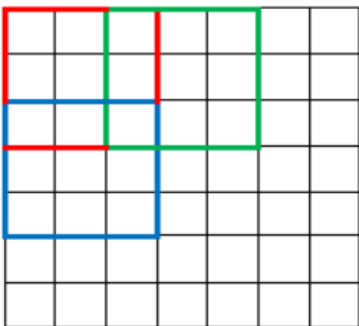


5 x 5 Output Volume

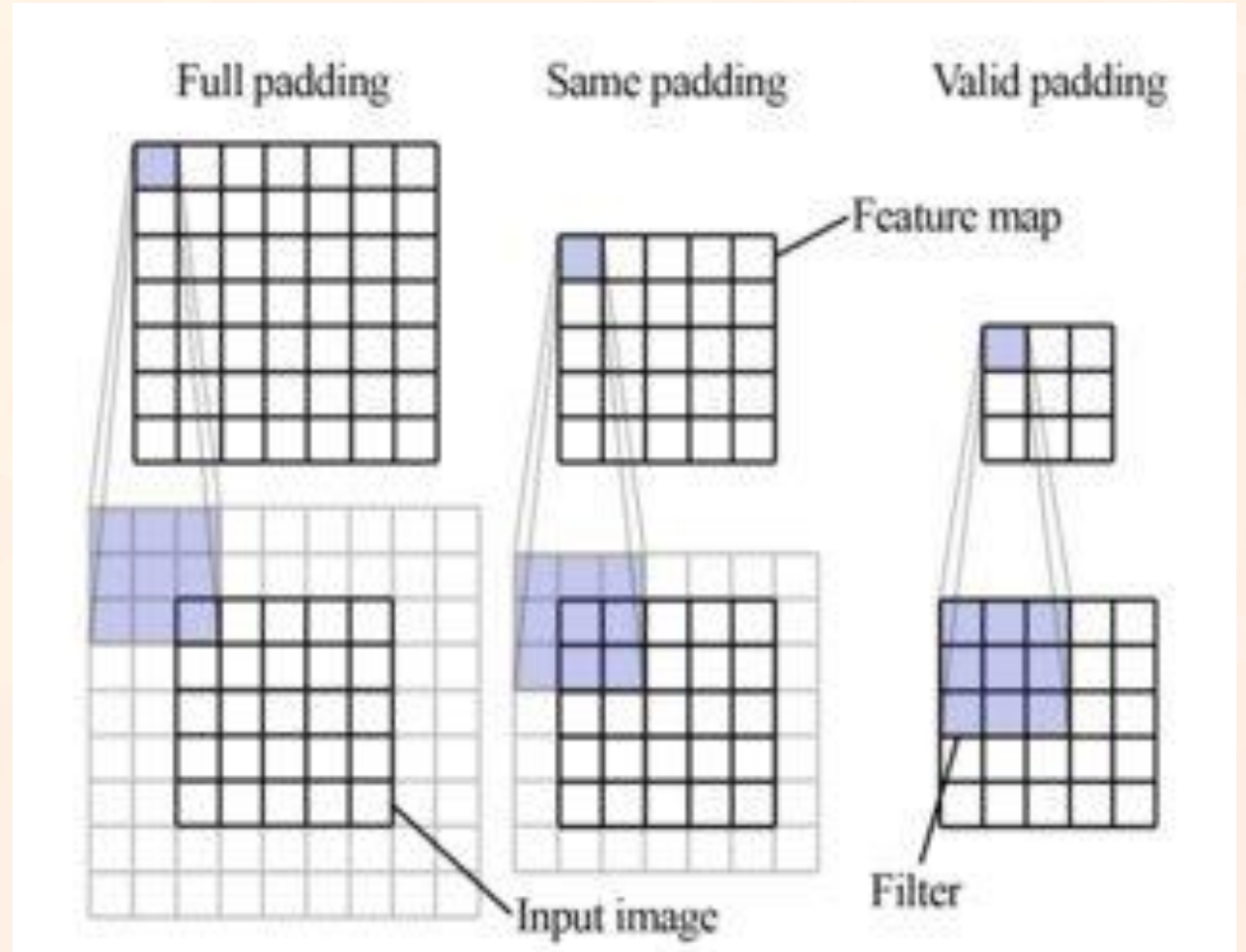


Stride de (2, 2) = 2

7 x 7 Input Volume



3 x 3 Output Volume



Exemple



2	3	3	4	7	3	4	4	6	3	2	4	9	4
6	1	6	0	9	1	8	0	7	1	4	0	3	2
3	-3	4	4	8	-3	3	4	8	-3	9	4	7	4
7	1	8	0	3	1	6	0	6	1	3	0	4	2
4	-3	2	4	1	-3	8	4	3	-3	4	4	6	4
3	1	2	0	4	1	1	0	9	1	8	0	3	2
0	-1	1	0	3	-1	9	0	2	-1	1	0	4	3

Image 7x7

*
Stride (2, 2)
Padding 'VALID'

3	4	4
1	0	2
-1	0	3

Noyau 3x3

=

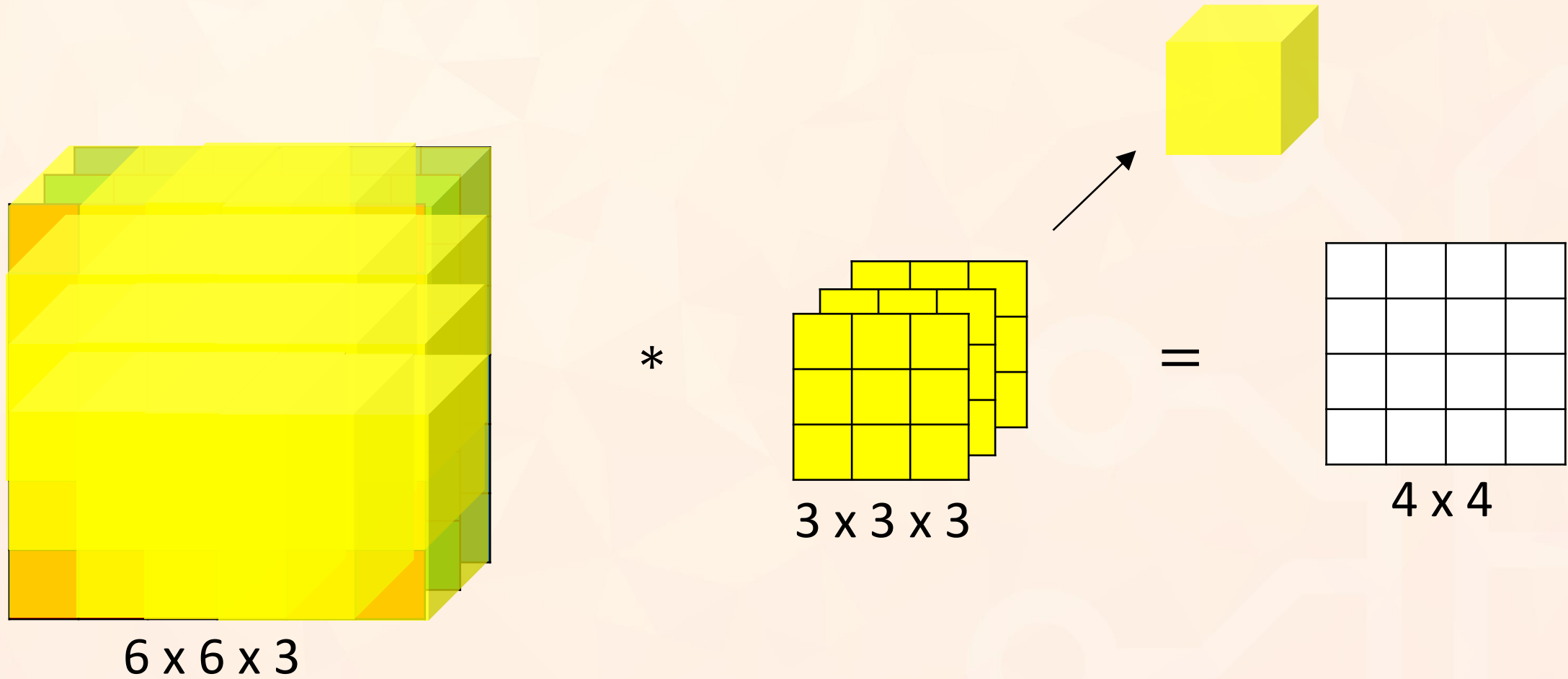
91	100	83
69	91	127
44	72	74

Feature map 3x3

$n \times n$ image $f \times f$ filter
padding p stride s

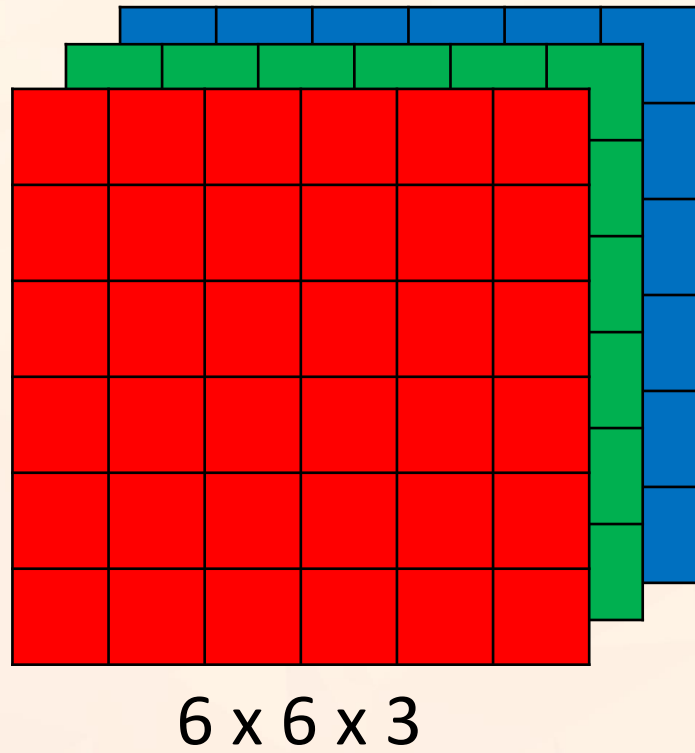
$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

Généralisation pour des volumes

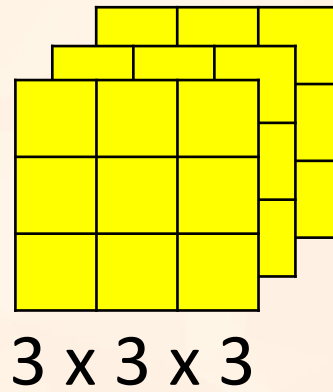


Important : Un noyau retourne toujours une seule 'feature map'

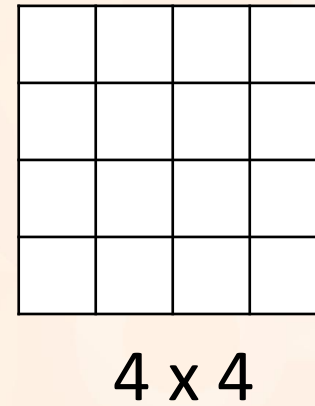
Plusieurs filtres



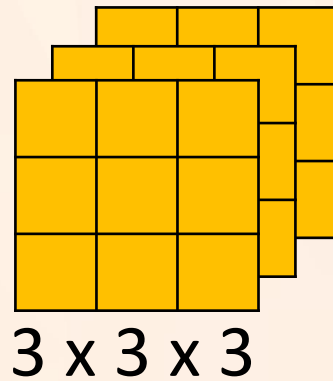
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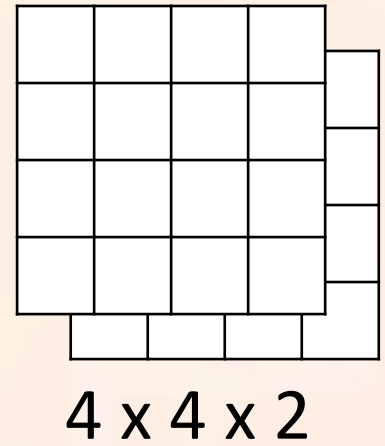
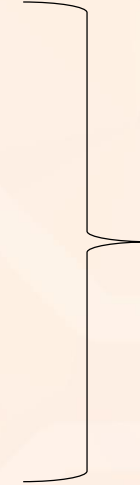
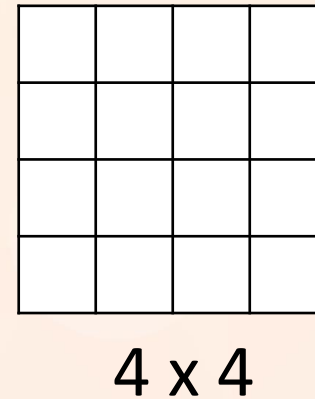
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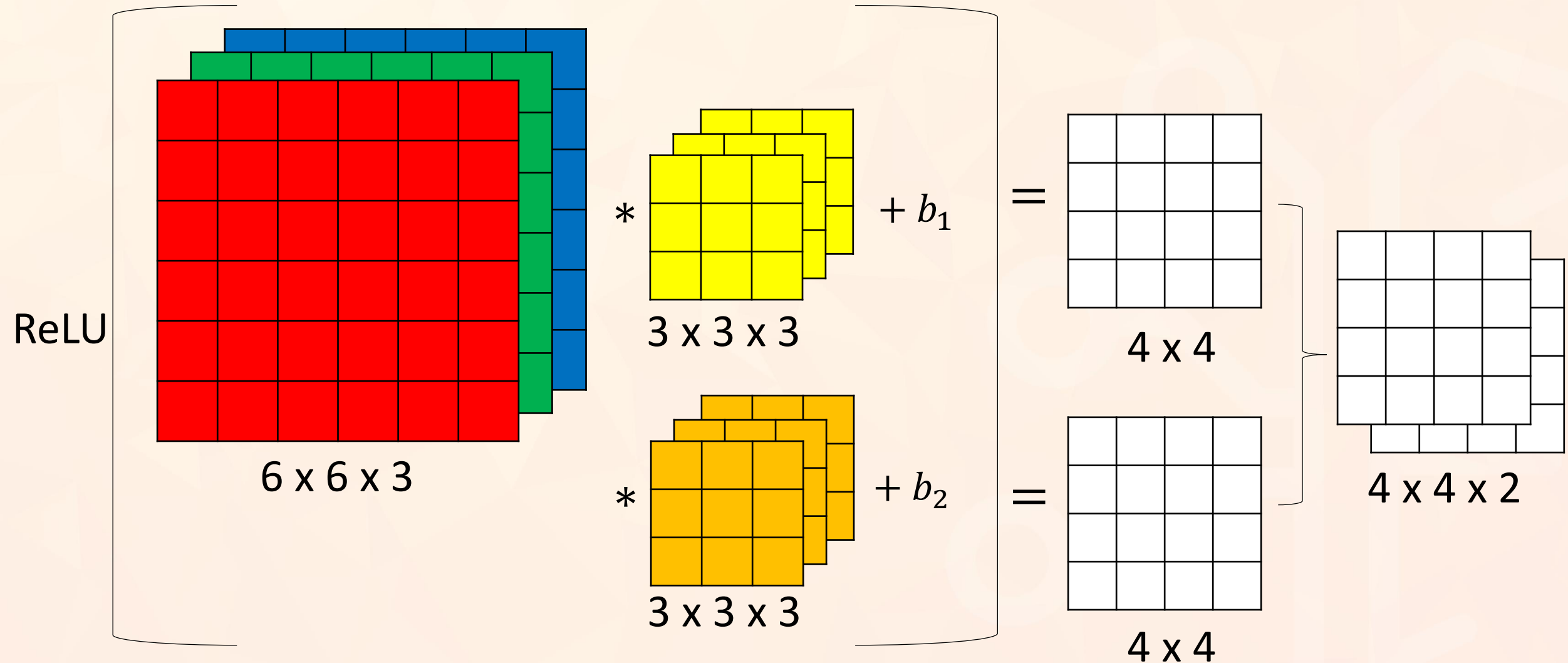
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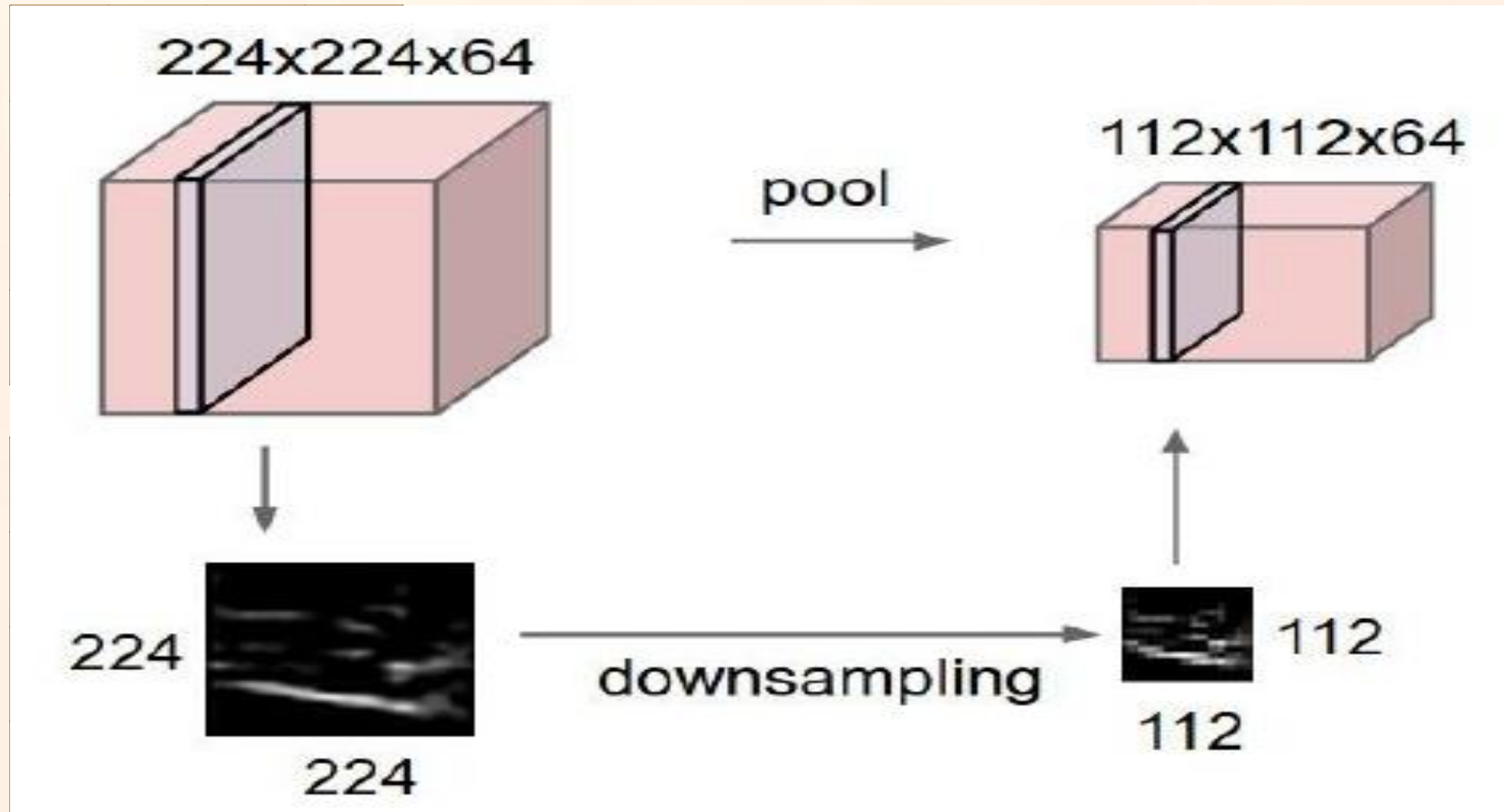
=



En fait...



Pooling: Réduction des dimensions



Example



1	3	2	1	3
2	9	7	1	5
1	4	3	8	2
8	3	6	1	0
5	6	1	2	9

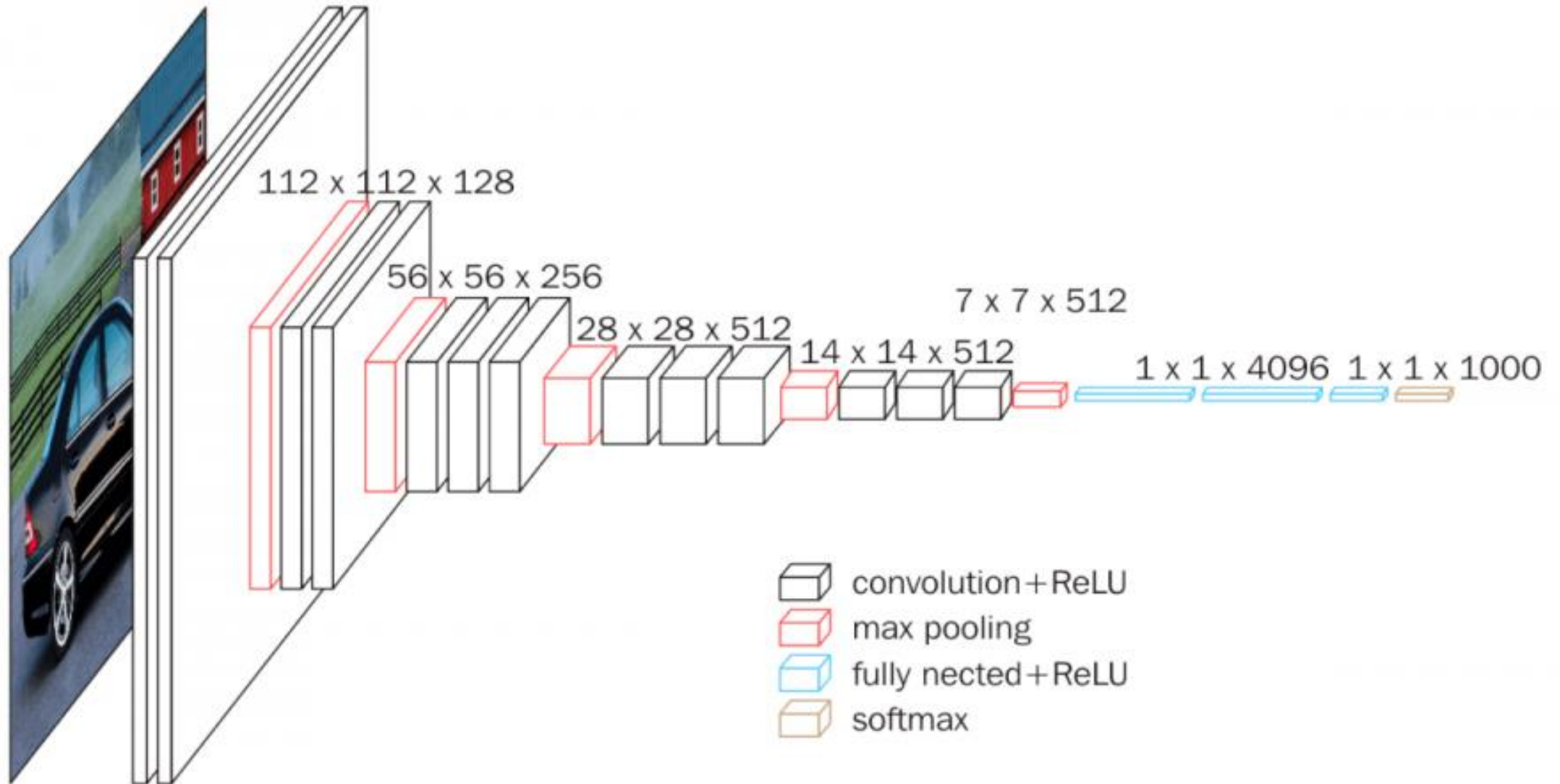
Max Pooling

→
Stride (1, 1)

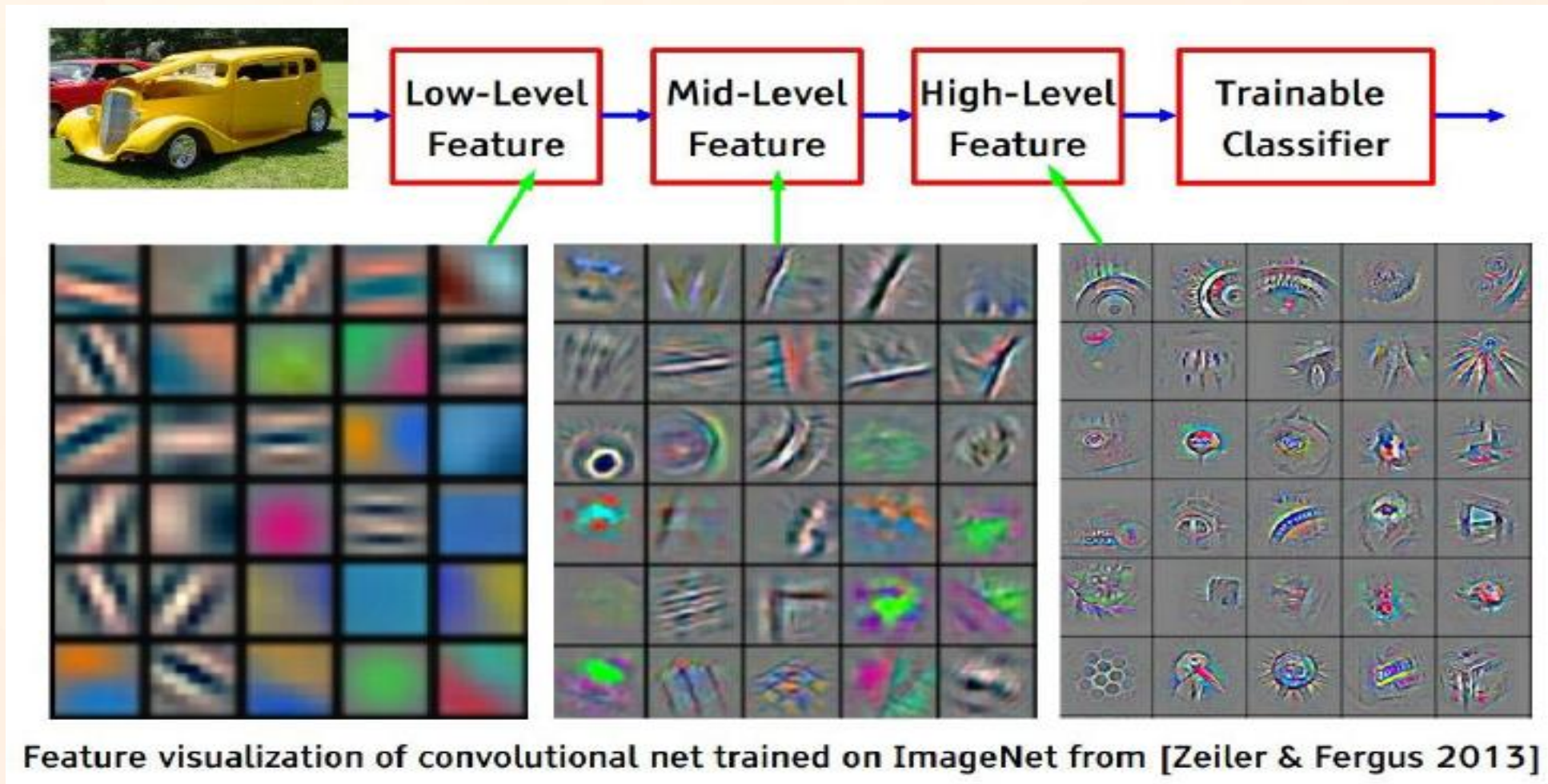
9	9	5
9	9	5
8	6	9

$$\left\lfloor \frac{n-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n-f}{s} + 1 \right\rfloor$$

Etude de cas: VGG19

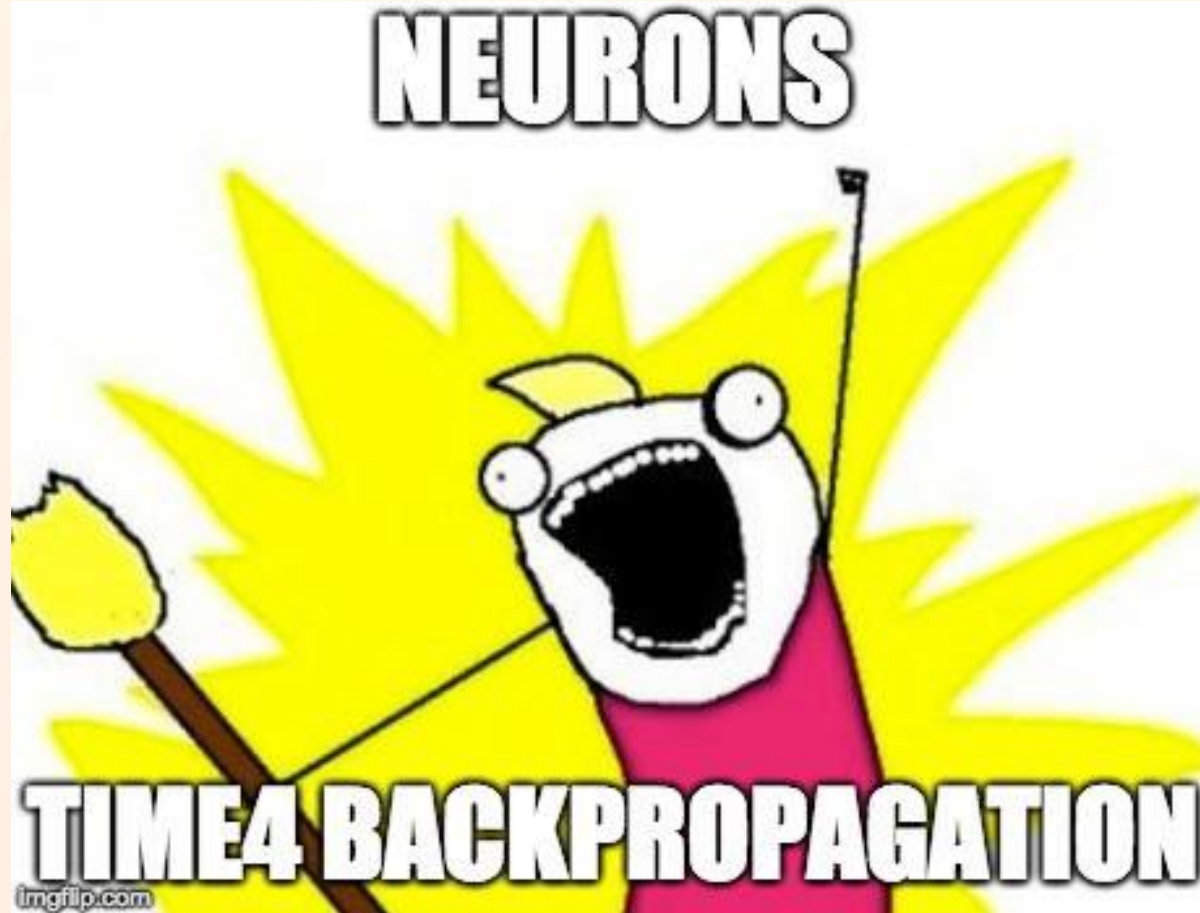


Vérification des hypothèses



https://www.youtube.com/watch?v=AgkflQ4IGaM&ab_channel=JasonYosinski

Now backpropagate



- Gradients de la sortie/cout par rapport aux poids/paramètres
- Graphe de computation
 - Règle de la chaine
 - Récursivité

Backpropagation 101: Notations



- Etant donné une fonction f et sa valeur en un point x , on veut « calculer » le gradient de f en x

$$f(x, y) = xy \quad \rightarrow \quad \frac{\partial f}{\partial x} = y \quad \frac{\partial f}{\partial y} = x$$

$$x=4, y=-3 \quad f(x, y) = -12$$

$$f(x, y) = x + y \quad \rightarrow \quad \frac{\partial f}{\partial x} = 1 \quad \frac{\partial f}{\partial y} = 1$$

$$x=4, y=-3 \quad f(x, y) = -1$$

$$f(x, y) = \max(x, y) \quad \rightarrow \quad \frac{\partial f}{\partial x} = 1(x \geq y) \quad \frac{\partial f}{\partial y} = 1(y \geq x)$$

Backpropagation 101: Graphe de computation



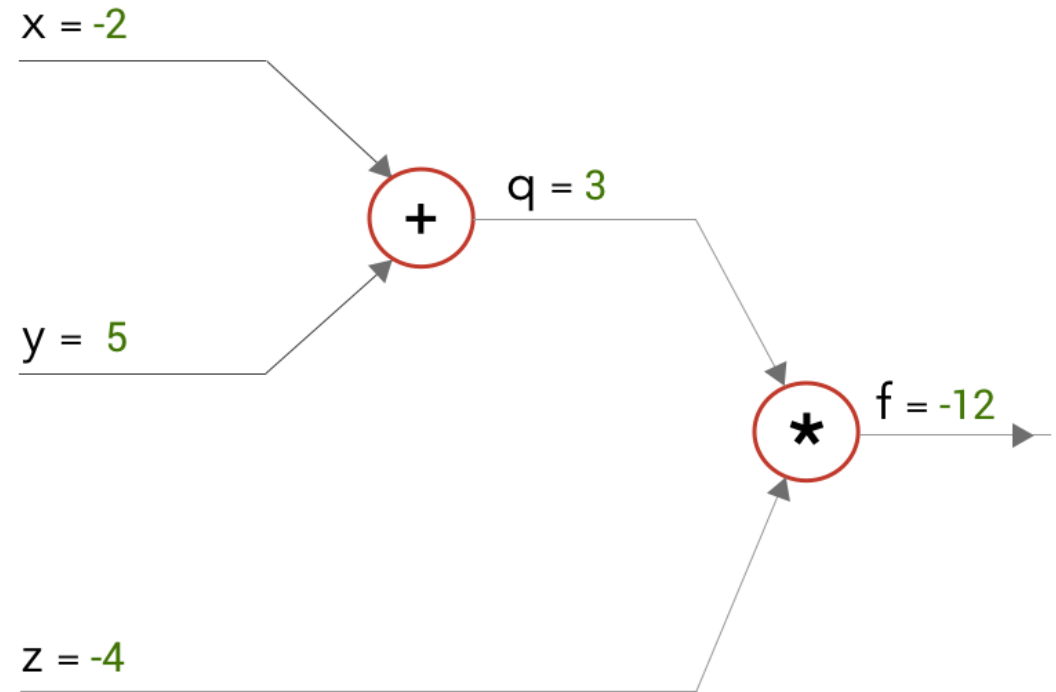
- Un exemple simple: $f(x, y, z) = (x + y) * z$

$$f(x, y, z) = (x + y)z$$

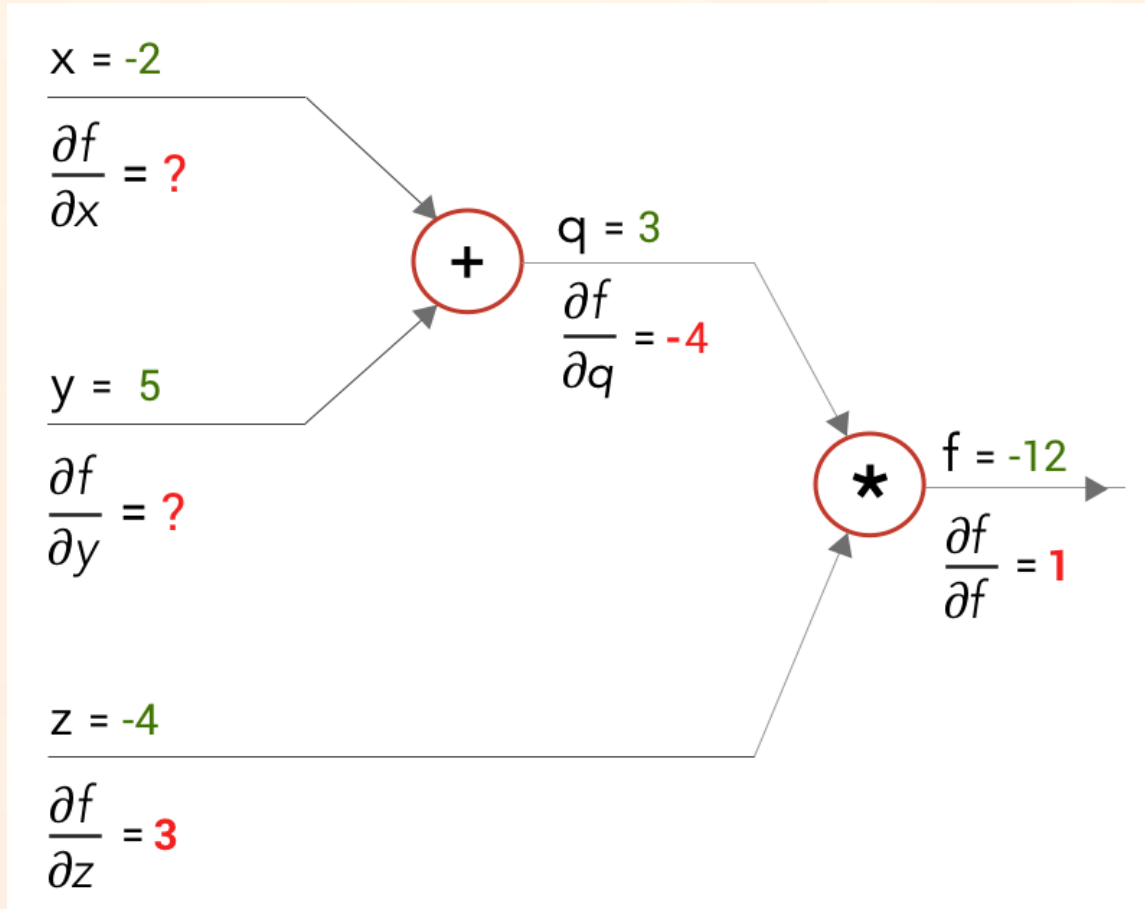
$$q = x + y$$

$$f = q * z$$

- $x = -2, y = 5, z = 4.$



Backpropagation 101: calcul des gradients



$$f = q * z$$

$$\frac{\partial f}{\partial q} = z \mid z = -4$$

$$\frac{\partial f}{\partial z} = q \mid q = 3$$

$$q = x + y$$

$$\frac{\partial q}{\partial x} = 1 \quad \frac{\partial q}{\partial y} = 1$$

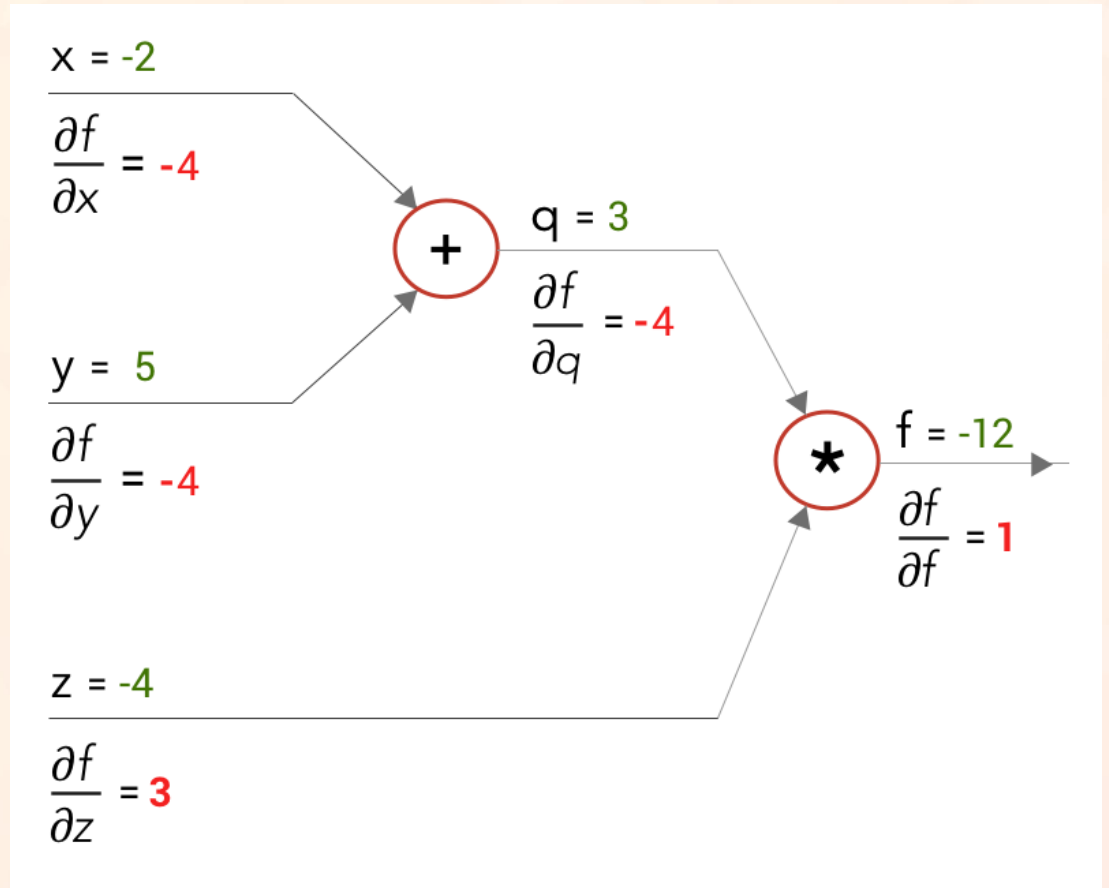
Backpropagation 101: Règle de la chaîne



$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} * \frac{\partial q}{\partial x}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} * \frac{\partial q}{\partial x} = -4 * 1 = -4$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} * \frac{\partial q}{\partial y} = -4 * 1 = -4$$



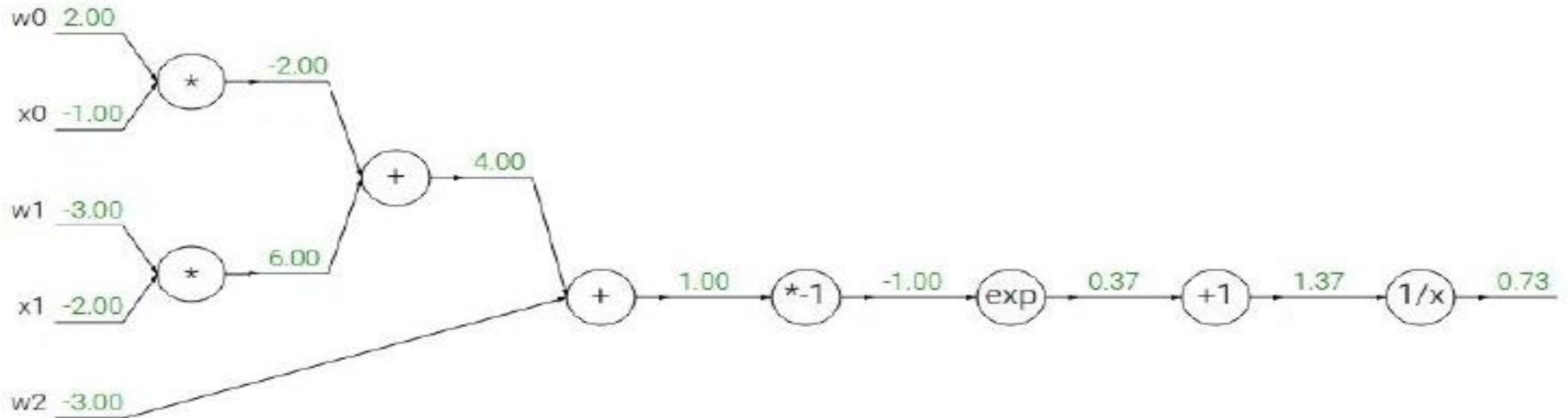
$$f(x, y, z) = (x + y) * z$$

Backpropagation 101: réseau de neurones

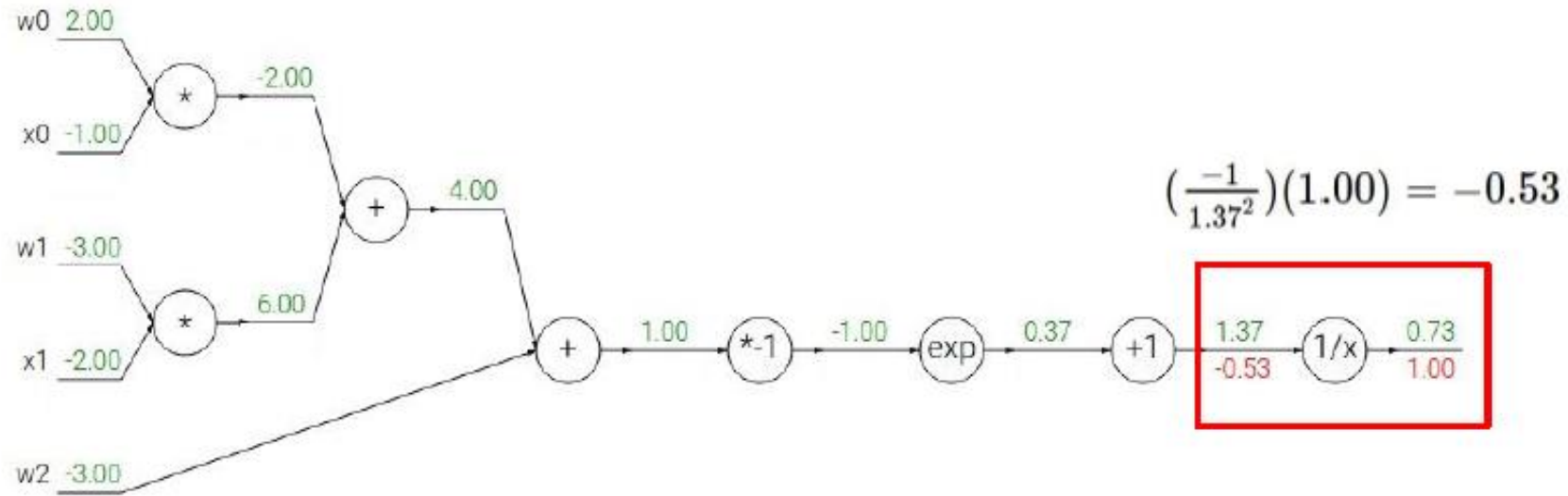


$$f(w, x) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$

$f(x) = e^x$	$\rightarrow$	$\frac{df}{dx} = e^x$		$f(x) = \frac{1}{x}$	$\rightarrow$	$\frac{df}{dx} = -1/x^2$
$f_a(x) = ax$	$\rightarrow$	$\frac{df}{dx} = a$		$f_c(x) = c + x$	$\rightarrow$	$\frac{df}{dx} = 1$



Backpropagation 101: réseau de neurones



$$f(x) = e^x$$

$\rightarrow$

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

$\rightarrow$

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

$\rightarrow$

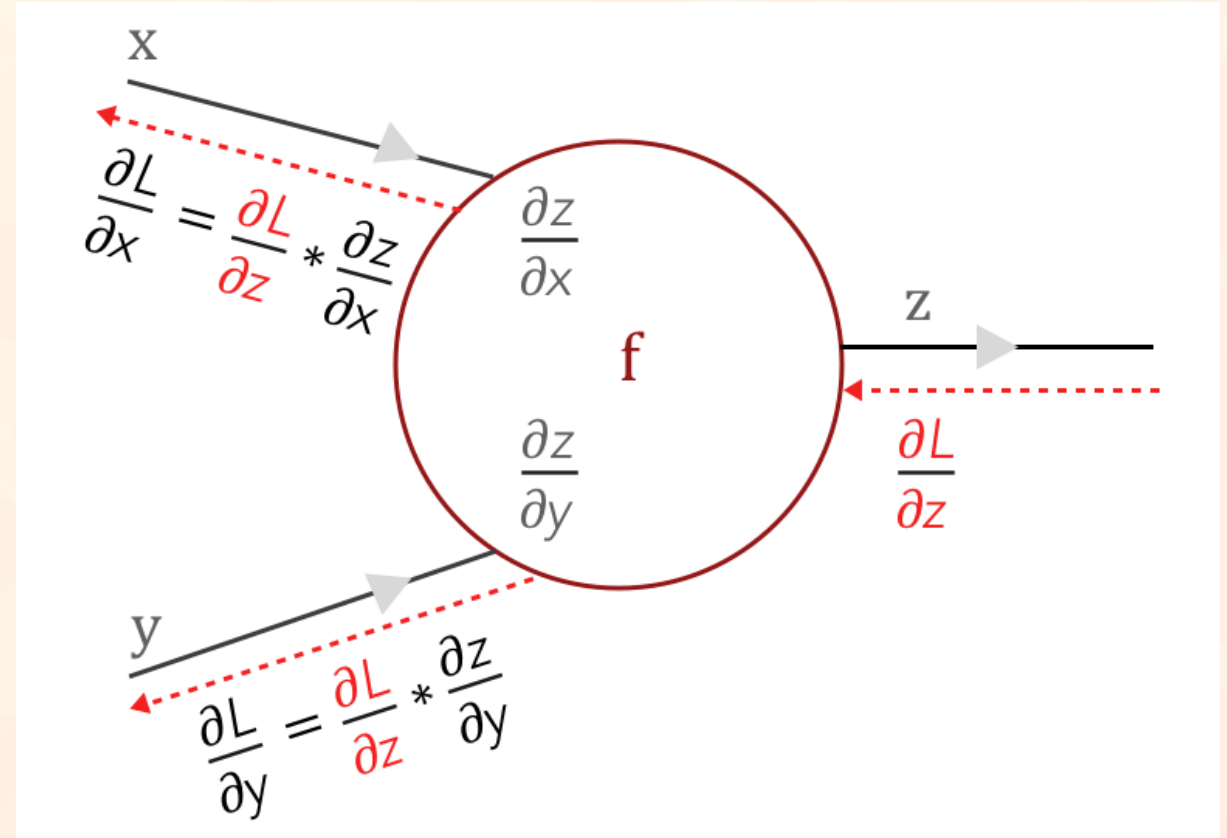
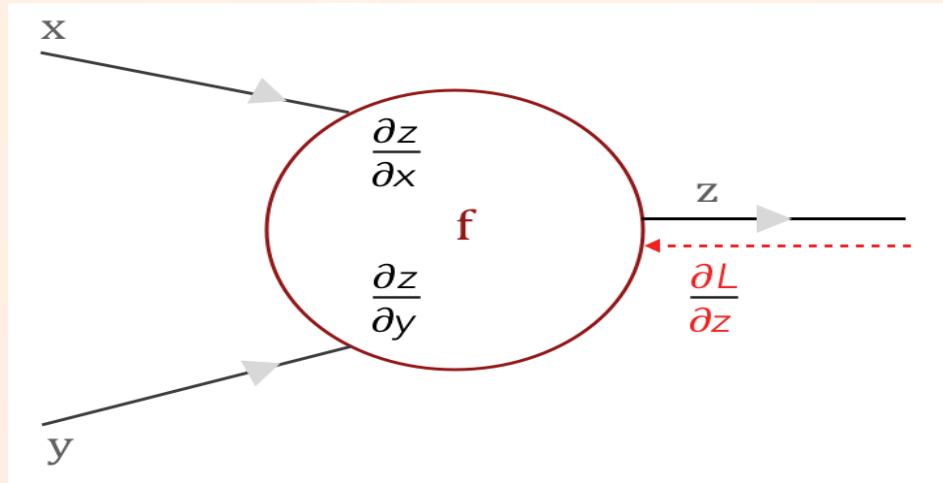
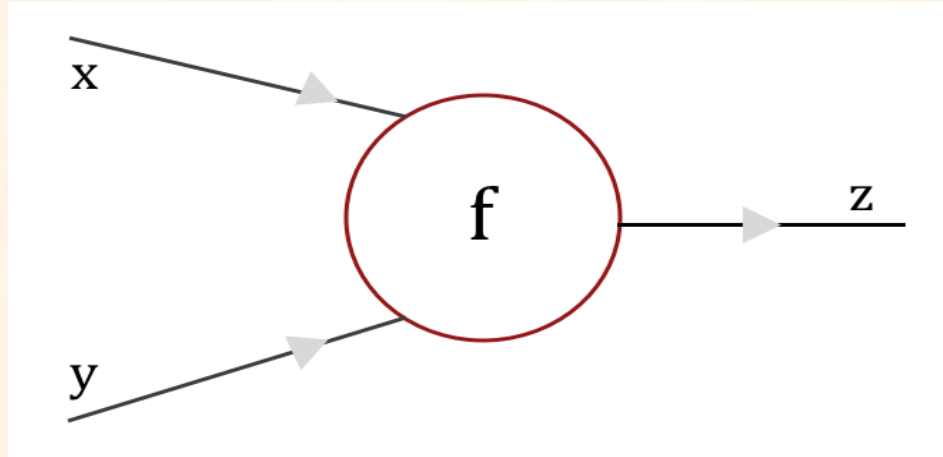
$$\frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x$$

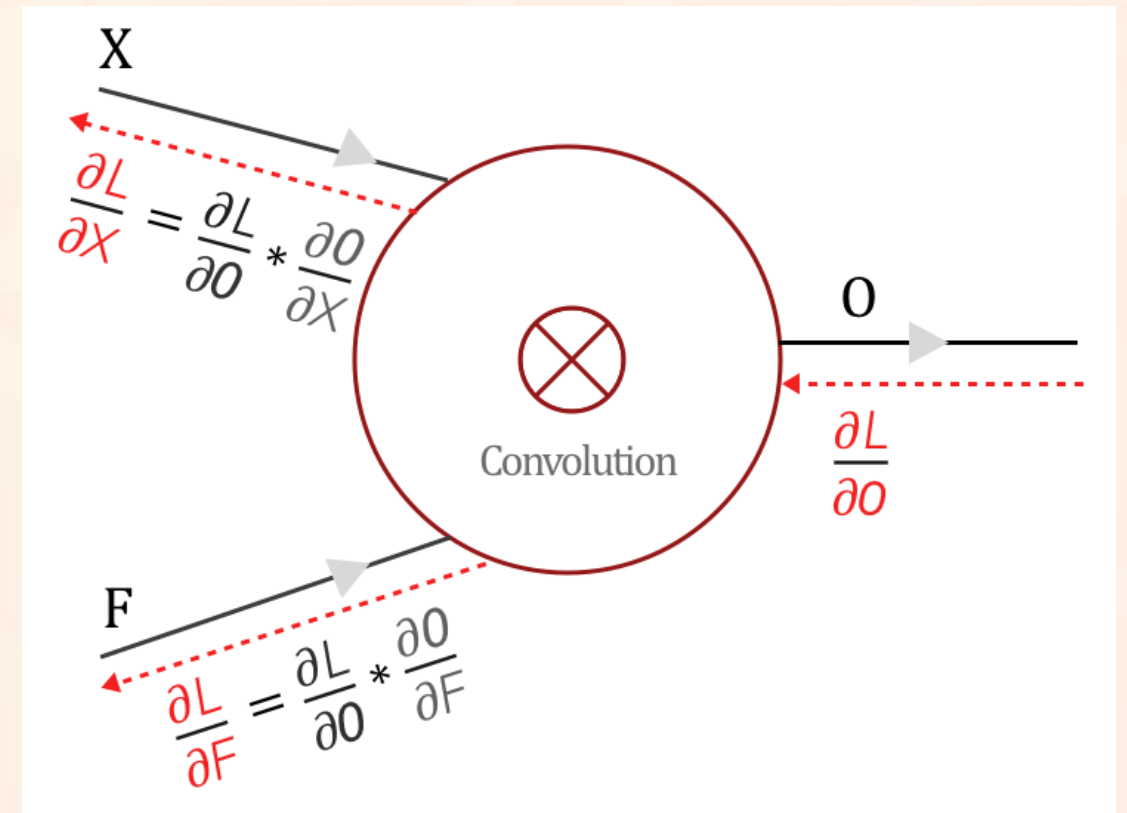
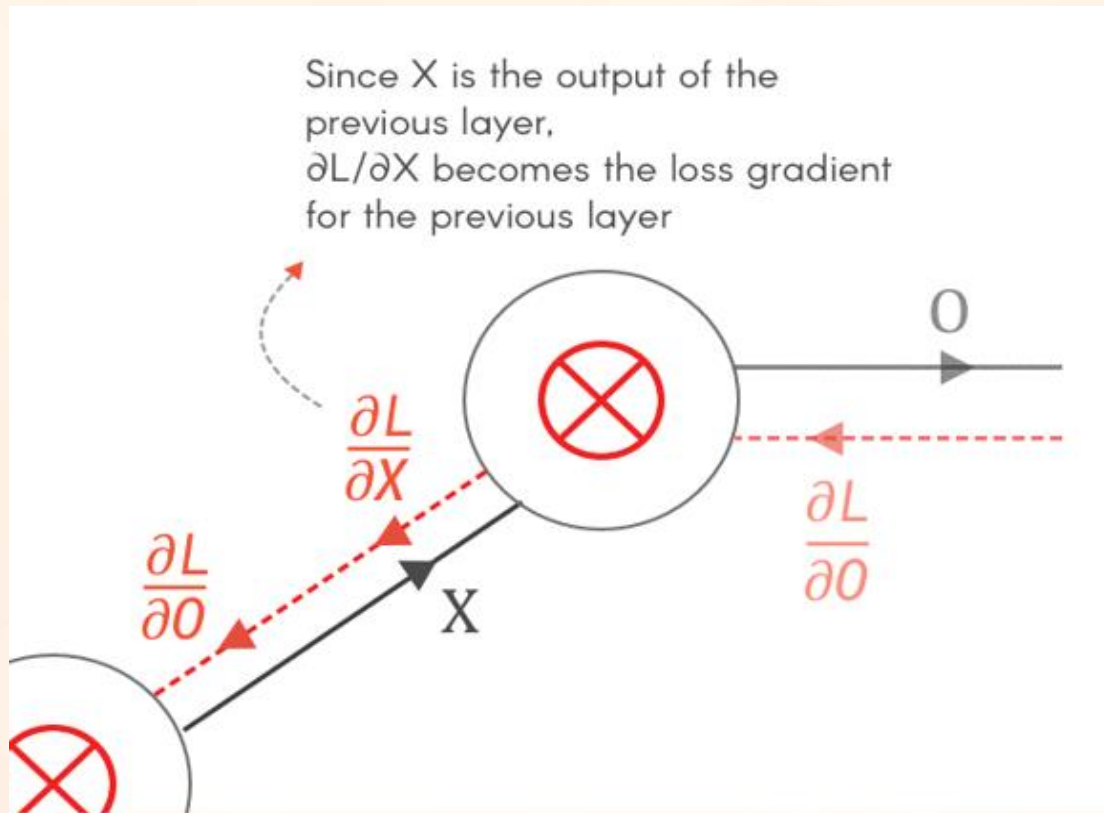
$\rightarrow$

$$\frac{df}{dx} = 1$$

Backpropagation 101: Plus généralement



Graph de convolution



Back-(to convolution)-propagation



X_{11}	X_{12}	X_{13}
X_{21}	X_{22}	X_{23}
X_{31}	X_{32}	X_{33}

Input **X**

F_{11}	F_{12}
F_{21}	F_{22}

Filter **F**

X_{11}	X_{12}	X_{13}
X_{21}	X_{22}	X_{23}
X_{31}	X_{32}	X_{33}

Input **X**



F_{11}	F_{12}
F_{21}	F_{22}

Filter **F**

$X_{11}F_{11}$	$X_{12}F_{12}$	X_{13}
$X_{21}F_{21}$	$X_{22}F_{22}$	X_{23}
X_{31}	X_{32}	X_{33}

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

$$\begin{array}{|c|c|} \hline O_{11} & O_{12} \\ \hline O_{21} & O_{22} \\ \hline \end{array} = \text{Convolution} \left(\begin{array}{|c|c|c|} \hline X_{11} & X_{12} & X_{13} \\ \hline X_{21} & X_{22} & X_{23} \\ \hline X_{31} & X_{32} & X_{33} \\ \hline \end{array}, \begin{array}{|c|c|} \hline F_{11} & F_{12} \\ \hline F_{21} & F_{22} \\ \hline \end{array} \right)$$

Output **O**

Gradients du filtre



$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

$$\frac{\partial O_{11}}{\partial F_{11}} = X_{11} \quad \frac{\partial O_{11}}{\partial F_{12}} = X_{12} \quad \frac{\partial O_{11}}{\partial F_{21}} = X_{21} \quad \frac{\partial O_{11}}{\partial F_{22}} = X_{22}$$

$$\frac{\partial L}{\partial F_i} = \sum_{k=1}^M \frac{\partial L}{\partial O_k} * \frac{\partial O_k}{\partial F_i}$$

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * X_{11} + \frac{\partial L}{\partial O_{12}} * X_{12} + \frac{\partial L}{\partial O_{21}} * X_{21} + \frac{\partial L}{\partial O_{22}} * X_{22}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * X_{12} + \frac{\partial L}{\partial O_{12}} * X_{13} + \frac{\partial L}{\partial O_{21}} * X_{22} + \frac{\partial L}{\partial O_{22}} * X_{23}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * X_{21} + \frac{\partial L}{\partial O_{12}} * X_{22} + \frac{\partial L}{\partial O_{21}} * X_{31} + \frac{\partial L}{\partial O_{22}} * X_{32}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * X_{22} + \frac{\partial L}{\partial O_{12}} * X_{23} + \frac{\partial L}{\partial O_{21}} * X_{32} + \frac{\partial L}{\partial O_{22}} * X_{33}$$

Gradient du filtre



$$\begin{array}{|c|c|} \hline \frac{\partial L}{\partial F_{11}} & \frac{\partial L}{\partial F_{12}} \\ \hline \frac{\partial L}{\partial F_{21}} & \frac{\partial L}{\partial F_{22}} \\ \hline \end{array} = \text{Convolution} \left(\begin{array}{|c|c|c|} \hline X_{11} & X_{12} & X_{13} \\ \hline X_{21} & X_{22} & X_{23} \\ \hline X_{31} & X_{32} & X_{33} \\ \hline \end{array}, \begin{array}{|c|c|} \hline \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \hline \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \\ \hline \end{array} \right)$$

where

$$\begin{array}{|c|c|c|} \hline X_{11} & X_{12} & X_{13} \\ \hline X_{21} & X_{22} & X_{23} \\ \hline X_{31} & X_{32} & X_{33} \\ \hline \end{array} = \text{Input X} \quad \begin{array}{|c|c|} \hline \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \hline \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \\ \hline \end{array} = \frac{\partial L}{\partial O} \quad \begin{array}{l} \text{Loss gradient} \\ \text{from previous} \\ \text{layer} \end{array}$$

Gradient de la couche précédente



$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

$$\frac{\partial O_{11}}{\partial X_{11}} = F_{11} \quad \frac{\partial O_{11}}{\partial X_{12}} = F_{12} \quad \frac{\partial O_{11}}{\partial X_{21}} = F_{21} \quad \frac{\partial O_{11}}{\partial X_{22}} = F_{22}$$

$$\frac{\partial L}{\partial X_i} = \sum_{k=1}^M \frac{\partial L}{\partial O_k} * \frac{\partial O_k}{\partial X_i}$$

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial O_{11}} * F_{11}$$

$$\frac{\partial L}{\partial X_{12}} = \frac{\partial L}{\partial O_{11}} * F_{12} + \frac{\partial L}{\partial O_{12}} * F_{11}$$

$$\frac{\partial L}{\partial X_{13}} = \frac{\partial L}{\partial O_{12}} * F_{12}$$

$$\frac{\partial L}{\partial X_{21}} = \frac{\partial L}{\partial O_{11}} * F_{21} + \frac{\partial L}{\partial O_{21}} * F_{11}$$

$$\frac{\partial L}{\partial X_{22}} = \frac{\partial L}{\partial O_{11}} * F_{22} + \frac{\partial L}{\partial O_{12}} * F_{21} + \frac{\partial L}{\partial O_{21}} * F_{12} + \frac{\partial L}{\partial O_{22}} * F_{11}$$

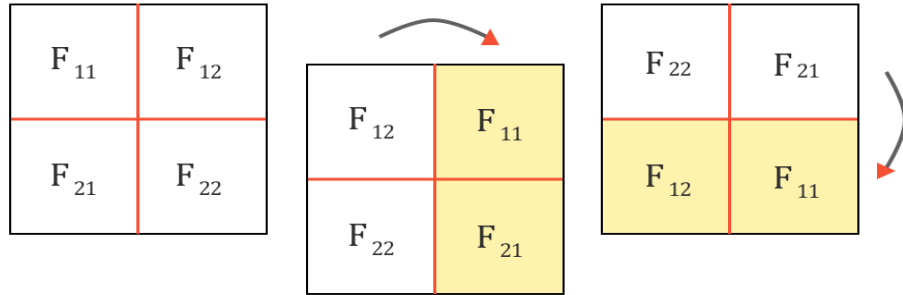
$$\frac{\partial L}{\partial X_{23}} = \frac{\partial L}{\partial O_{12}} * F_{22} + \frac{\partial L}{\partial O_{22}} * F_{12}$$

$$\frac{\partial L}{\partial X_{31}} = \frac{\partial L}{\partial O_{21}} * F_{21}$$

$$\frac{\partial L}{\partial X_{32}} = \frac{\partial L}{\partial O_{21}} * F_{22} + \frac{\partial L}{\partial O_{22}} * F_{21}$$

$$\frac{\partial L}{\partial X_{33}} = \frac{\partial L}{\partial O_{22}} * F_{22}$$

Gradient de la couche précédente



F_{22}	F_{21}
F_{12}	F_{11}

Filter F

$\frac{\partial L}{\partial o_{11}}$	$\frac{\partial L}{\partial o_{12}}$
$\frac{\partial L}{\partial o_{21}}$	$\frac{\partial L}{\partial o_{22}}$

Loss Gradient $\frac{\partial L}{\partial o}$

$$\frac{\partial L}{\partial x_{11}} = F_{11} * \frac{\partial L}{\partial o_{11}}$$

F_{22}	F_{21}	
F_{12}	$F_{11} \frac{\partial L}{\partial o_{11}}$	$\frac{\partial L}{\partial o_{12}}$
	$\frac{\partial L}{\partial o_{21}}$	$\frac{\partial L}{\partial o_{22}}$

@pavisj

$\frac{\partial L}{\partial x_{11}}$	$\frac{\partial L}{\partial x_{12}}$	$\frac{\partial L}{\partial x_{13}}$
$\frac{\partial L}{\partial x_{21}}$	$\frac{\partial L}{\partial x_{22}}$	$\frac{\partial L}{\partial x_{23}}$
$\frac{\partial L}{\partial x_{31}}$	$\frac{\partial L}{\partial x_{32}}$	$\frac{\partial L}{\partial x_{33}}$

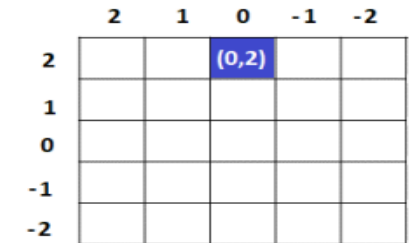
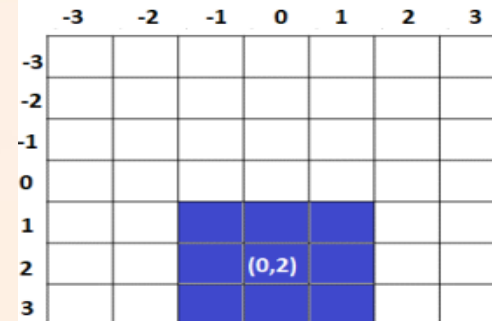
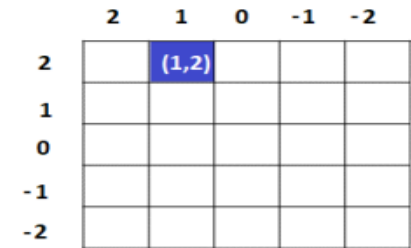
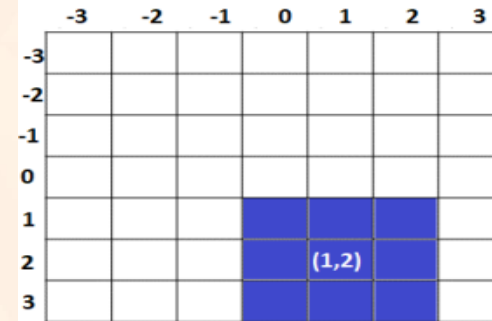
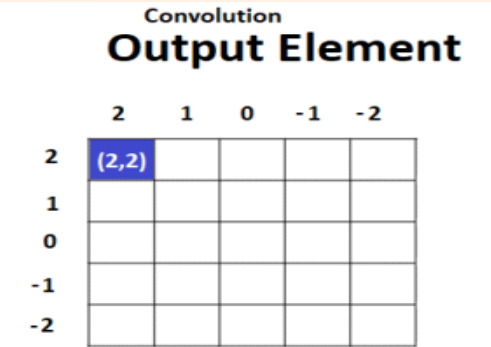
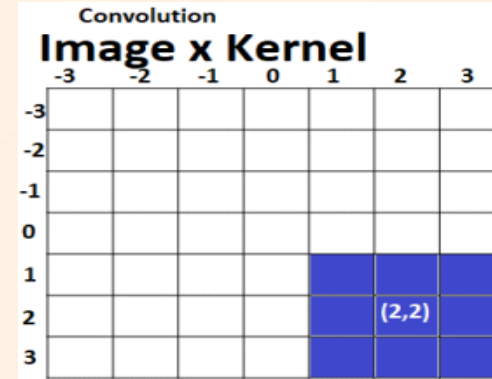
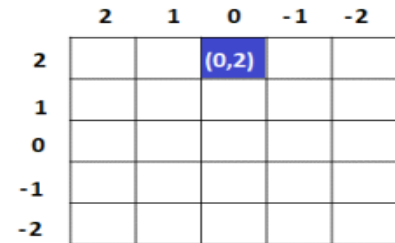
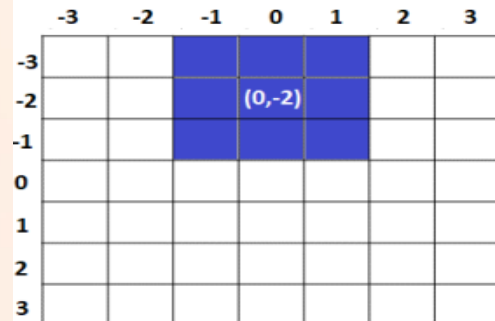
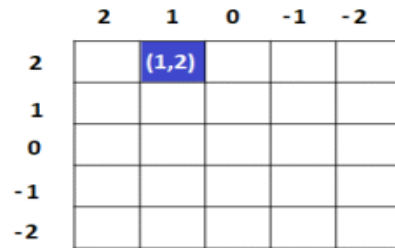
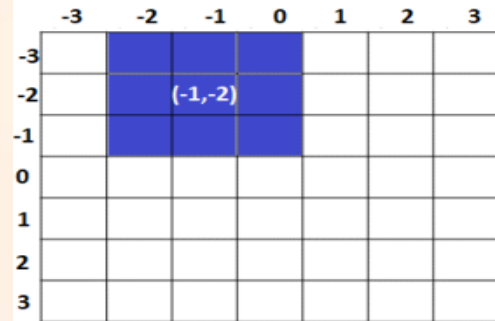
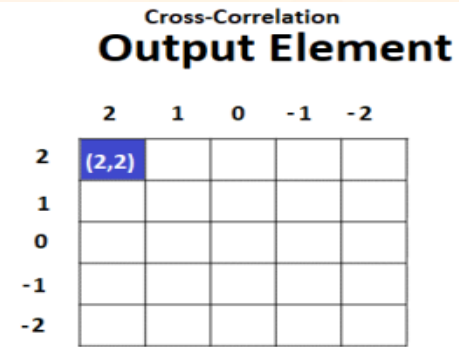
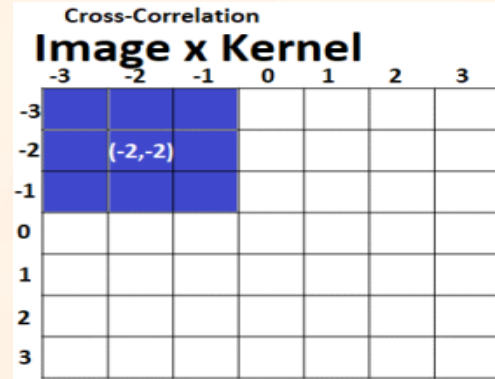
$\frac{\partial L}{\partial x}$

$$= \text{Full Convolution} \left(\begin{array}{|c|c|} \hline F_{22} & F_{21} \\ \hline F_{12} & F_{11} \\ \hline \end{array}, \begin{array}{|c|c|} \hline \frac{\partial L}{\partial o_{11}} & \frac{\partial L}{\partial o_{12}} \\ \hline \frac{\partial L}{\partial o_{21}} & \frac{\partial L}{\partial o_{22}} \\ \hline \end{array} \right)$$

Filter F

Loss Gradient $\frac{\partial L}{\partial o}$

Convolution vs Cross-Correlation



Finalement



$$\frac{\partial L}{\partial F} = \text{Convolution} \left(\text{Input } X, \text{ Loss gradient } \frac{\partial L}{\partial O} \right)$$

$$\frac{\partial L}{\partial X} = \text{Full Convolution} \left(\begin{array}{c} 180^\circ \text{rotated} \\ \text{Filter } F \end{array}, \text{ Loss Gradient } \frac{\partial L}{\partial O} \right)$$

